Equilibrium strategies for forest restoration via an approach based on dynamic games

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Abstract. It is investigated whether forest restoration should be the responsibility of the forest owners or the non-owners from both social welfare and environmental perspectives. Two types of countries are considered: one where the forest owner profits from deforestation - the “deforester” - and another where the forest non-owner generates income through production – the “producer”. Both face costs due to the negative externalities caused by pollution. They participate in a non-cooperative dynamic game where the restoration efforts are alternatively carried out by either the deforester or the producer. By using the cooperative mode of play as a benchmark, the economic and environmental effects of the two non-cooperative modes are analyzed.
Keywords: pollution accumulation, environmental absorption efficiency, emissions, deforestation, restoration, non-cooperative strategies.

1. Introduction

The environmental impact of deforestation is twofold. First, it reduces the carbon storage efficiency in the biosphere, thereby causing an increase in the carbon stock in the atmosphere (e.g., Canadell and Raupach 2008). Second, it also acts as a non-negligible carbon source (e.g., Baccini et al. 2012).

Prior to a G7 summit in 2019[1], French President Emmanuel Macron stated that “a genuine ecocide is unfolding throughout the Amazon, not solely in Brazil.” Consequently, a dispute arose between Brazil and France concerning the handling of the Amazon rainforest. This raises the issue of the governance of forests among owners and non-owners. Inspired by this recent controversy, we investigate from both social welfare and environmental viewpoints whether the restoration of forests should be undertaken by the forest owners or non-owners.

To date, the economic literature has largely treated pollution control and deforestation as two separate issues (see, e,g, El Ouardighi et al. 2020, El Ouardighi 2025). The current work differs in that it accounts for the basic, empirically established, influences that prevail between pollution accumulation and deforestation.

2. The dynamic game model

We consider two countries: One involved in the production of economic goods, i.e., a producer; and the other that owns and exploits forests, i.e., a deforester. Both activities, i.e., production and deforestation, generate polluting emissions. The emissions resulting from the production rate are denoted by $u(t) \geq 0$. The deforestation rate, defined by $v(t) \geq 0$, generates polluting emissions at a proportional rate: $\alpha v(t)$, where $\alpha > 0$. The evolution of the pollution stock, denoted by $P(t) \geq 0$, is described as

$$\dot{P}(t) = u(t) + \alpha v(t) - A(t)P(t), \qquad P(0) = P_0 \geq 0 \qquad (1)$$

In (1), the pollution stock decreases with the environmental absorption efficiency rate, $A(t)$, which obeys the transition equation:

$$\dot{A}(t) = w(t)A(t) - v(t) - \gamma P(t), A(0) = A_0 \geq 0 \qquad (2)$$

where $\gamma > 0$. According to (2), the environmental absorption efficiency decreases with deforestation, $v(t) \geq 0$, and with the destructive impact of the pollution stock, $\gamma P(t)$. The negative impact of $v(t)$ illustrates the fact that deforestation reduces the efficiency of carbon sinks. The negative influence of $\gamma P(t)$, $\gamma > 0$, reflects the hysteretic nature of the destructive impact of the pollution stock on the biosphere. However, when $A(t) > 0$, the environmental absorption efficiency increases with a restoration effort, if any, denoted by $w(t) \geq 0$. We assume in (2) that the marginal effectiveness of restoration efforts depends on the absorption efficiency in a linear way.

The producer is supposed to draw revenues from production-based emissions according to a linear-quadratic revenue function $u(t)\left(a - \frac{u(t)}{2}\right)$, where $a > 0$ is the revenue-maximizing production level and $u(t) \leq$

1 https://www.reuters.com/article/business/environment/frances-macron-says-real-ecocide-going-on-in-amazon-idUSKCN1VD2AM/

$2a$. The deforester draws revenues from deforestation, with a revenue function $v(t)\left(b - \frac{v(t)}{2}\right)$, where $b > 0$ is the revenue-maximizing deforestation level and $v(t) \leq 2b$. The negative externalities of the pollution stock are valued as increasing convex function of pollution, $\frac{cP(t)^2}{2}$, $c > 0$. We consider two scenarios, namely that the environmental absorption efficiency is restored either by the deforester or by the producer, to determine which of these scenarios results in the most conservationist long-run outcome. The producer's interest in restoring the absorption efficiency is that it can maintain its revenues from the production of economic goods. Similarly, by allowing the producer to undertake the restoration effort, the deforester is able to preserve its revenues from deforestation. Finally, both players are able to reduce their pollution costs. Due to decreasing returns, the restoration efforts generate an increasing quadratic cost, denoted by $\frac{\mathrm{d}w(t)^2}{2}$, $\mathrm{d} > 0$. Assuming an infinite planning horizon, and denoting the discounting rate by $r > 0$, the non-cooperative problems in the case where the players are unable to cooperate write:

$$\operatorname{Max} W^p = \int_0^\infty e^{-rt}\left[u(t)\left(a - \frac{u(t)}{2}\right) - \frac{h_1 w(t)^2}{2} - \frac{cP(t)^2}{4}\right] \mathrm{d}t$$

$$\operatorname{Max} W^d = \int_0^\infty e^{-rt}\left[v(t)\left(b - \frac{v(t)}{2}\right) - \frac{h_2 w(t)^2}{2} - \frac{cP(t)^2}{4}\right] \mathrm{d}t$$

where 'p' and 'd' resp. stand for producer (forest non-owner) and deforester (forest owner), under (1)-(2), with W^p and W^d denoting the producer's and deforester's welfares. Here, $(h_1, h_2) = (0,1)$ if the restoration is done by the deforester, $(h_1, h_2) = (1,0)$ in the converse case.

3. Analysis and outcomes

At the initial period, the players have to decide which scenario should apply (i.e., who restores) throughout the game. Whereas the restorer is committed to restore in any case, the non-restorer has to decide whether to adopt a commitment- (open-loop) or contingent-based (closed-loop) Nash equilibrium strategy, called homogeneous and heterogeneous mode of play, resp. The former allows for strategic interaction at the initial instant of time only, whereas the latter involves strategic interaction throughout the game because the non-restorer's decisions are contingent on the current state of the game.

- We determine analytically the equilibrium conditions related to the two kinds of strategies.
- Using the cooperative equilibrium as a benchmark, we assess numerically how pollution, environmental absorption efficiency and social welfare are affected by the choice of restorer, resp. for the cooperative and the two non-cooperative modes of play.
- We compare the different solutions in terms of both the steady states and their associated transient path, via the numerical solution of boundary value problems.
- For each mode of play and each scenario, we characterize the whole spectrum of solutions in a wide range of the parameter space for the key parameters.
- We get information on the relative merits of the deforester as restorer versus the producer as restorer.

A flavor of the numerical analysis is provided in Table 1, where $W = W^d + W^p$ is the overall social welfare and the superscript ∞ denotes steady state. Our results suggest that when homogeneous mode of play with commitment-based strategies prevails, it does not matter from an environmental viewpoint whether the forest restoration efforts are made by the forest owner, i.e., the deforester, or non-owner, i.e., the producer. This result is proved analytically both for the steady states and the transient paths, and verified numerically, regardless of whether the revenue-maximizing production level of the producer is greater or lower than the revenue-maximizing deforestation level of the deforester. From an economic viewpoint, however, it is not in the producer's best interest to undertake the forest restoration, regardless of whether the mode of play is homogeneous or heterogeneous. The decision of who among the forest owner and non-owner should act as a forest restorer should take into account three essential factors: the non-owner's relative potential revenue, the mode of play and the initial state of the forests.

Acknowledgements

The authors were supported by the 2019 Italy-France Galileo project (IT-G19-48, FR-42089QH). M. Sanguineti was supported by the Italian Ministry of Research (PRIN project 2022S8XSMY), the Ministry of Enterprises and Made in Italy (project F/310027/02/X56), the FISA-2022-00827 project "UAVFIRE", and CNR (PDGP DIT.AD021.104 project). G,. Gnecco and M. Sanguineti are members of GNAMPA-INdAM.

Table 1. Example of numerical results: steady state values and overall social welfare for $(a, b) = (0.4,0.1)$

Mode of play	P^∞	A^∞	u^∞	v^∞	w^∞	W^{d^∞}	W^{p^∞}	W^∞	W^d	W^p	W
Cooperative	0.1734	2.3726	0.3989	0.0835	0.0388	-	-	1.67916	-	-	1.68112
Homogeneous (deforester restores)	0.1893	2.1727	0.3991	0.0808	0.0415	0.07726	1.59820	1.67546	0.08235	1.59871	1.68106
Homogeneous (producer restores)	0.1893	2.1727	0.3991	0.0808	0.0415	0.09450	1.58095	1.67546	0.09589	1.58516	1.68106
Heterogeneous (deforester restores)	0.1894	2.1737	0.3996	0.0808	0.0415	0.07727	1.59820	1.67547	0.08235	1.5987	1.68105
Heterogeneous (producer restores)	0.2873	1.3932	0.3985	0.0113	0.0184	0.01715	1.59246	1.60961	0.06249	1.5959	1.6584

References

Canadell, J.G. and Raupach, M.R. (2008), Managing forests for climate change, *Science*, **320**, 1456-1457.

Baccini, A., Goetz, S.J., Walker, W.S., Laporte, N.T., Sun, M., Sulla-Menashe, D., Hackler J., Beck, P.S.A., Dubayah, R., Friedl, M.A., Samanta, S. and Houghton, R.A., 2012. Estimated carbon dioxide emissions from tropical deforestation improved by carbon-density maps, *Nature Climate Change* **2**, 182-185.

El Ouardighi, F. (2025), Essays on Pollution Control in Economics and Management Science – An Interdisciplinary View. Springer.

El Ouardighi, F., Kogan, K., Gnecco, G. and Sanguineti, M. (2020), Transboundary pollution control and environmental absorption efficiency management, *Annals of Operations Research*, **287**, 653-681.