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# Mathematics for Optimal Design of Sustainable Infrastructures

**VAZQUEZ-MENDEZ M.E.[1,2,*], ALVAREZ-VAZQUEZ L.J.[3], GARCIA-CHAN N.[4], MARTÍNEZ A.[3], RODRÍGUEZ C.[5]**

[1]CITMAga, 15782 Santiago de Compostela, Spain

[2]Departamento de Matemática Aplicada. Universidade de Santiago de Compostela. EPSE, 27002 Lugo, Spain

[3]Departamento de Matemática Aplicada II. Universidade de Vigo. EIT, 36310 Vigo, Spain

[4]Departamento de Física. Universidad de Guadalajara. CUCEI, 44430 Guadalajara, Mexico

[5]Departamento de Matemática Aplicada. Universidade de Santiago de Compostela. F. Matemáticas, 15782 Santiago, Spain

*corresponding author:
e-mail: miguelernesto.vazquez@usc.es

**Abstract** The main objective of this work is to show how mathematics, particularly the combination of modelling, numerical simulation and optimization, is a useful tool in the design of sustainable infrastructures. To do it, we clarify what we understand by systems optimization and present three interesting environmental problems (related to sustainable infrastructures) we have studied in the last decade and that fit well in this framework: the design of a river fishway to help migratory fish to climb a dam, the design of an irrigation channel to minimize sedimentation and erosion, and the management of an urban road network with an environmental perspective. Finally, we briefly discuss on some conclusions which can be drawn of this work.

**Keywords:** Sustainable Development Goals, Systems Optimization, Modelling, Numerical Simulation, Optimal Control

## 1. Introduction

In 2015, all United Nations Member States adopted the Agenda for Sustainable Development, which is a call for action to promote prosperity while protecting the planet. The core of this agenda is organized around 17 goals, named Sustainable Development Goals (SDG), which together collects 169 targets that are intended to be achieved by 2030 year (United Nations, 2023). As it could not be otherwise, this is a very hot topic in the current scientific literature and there are countless papers dealing with this subject. On March 22, 2023, the Scopus citation database had indexed 4671 papers with “Sustainable Development Goal” in the title, of which 99 were in the Mathematics subject area. For example, in the field of operational research (OR), a view of works focusing on the SDG can be seen in Smith (2019).

Within the framework of Mathematics, and specifically in the field of systems optimization, this paper focuses on SDG 9, which deals with Industry, Innovation and Infrastructure. To be precise, we center our attention exclusively on the first of its targets: “Develop quality, reliable, sustainable and resilient infrastructures”, and we will try to show how systems optimization can be a very useful tool to achieve this objective. To do this, we will explain what the systems optimization process consists of, from a mathematical point of view, and we will exemplify it in three problems related with the SDG 9.1, on which the authors have worked intensively in recent years.

## 2. Systems optimization

When faced with any real problem, regardless of the area in which it arises, it is necessary to predict (simulate) what will happen when a decision is taken. For example, if a barrier is going to be placed in the sea to contain hydrocarbon spills, the behavior of that barrier in different situations of waves must be previously studied, to know if it will be able, or not, to fulfill its task (Castro et al., 2010).

Two different techniques can be used to simulate a process, which should complement each other. On the one hand, an experimental model (for example, a scale model developed in the laboratory) can be used to provide experimental results of the behavior to be analyzed. The other option is *mathematical modelling*: formulating a system of equations (model), whose solution provides the values of the variables to be studied. Solving this model numerically (what is known as *numerical simulation*) provides a set of results which must be compared with those obtained experimentally to analyze whether, or not, the model is useful to predict the performance of the phenomenon under study. If the numerical results are not good, the model must be adjusted, solving it successively until the obtained results agree with the experimental ones. When that happens, the model is said to be validated, and it can already be used to predict the performance in similar situations. This is when the question arises: if you can predict what happens in different situations, why not design the best of them? The control of this entire process from the point of view of optimizing one or more criteria is what we understand by *systems optimization*. Figure 1 contains a diagram of the different stage of this process.

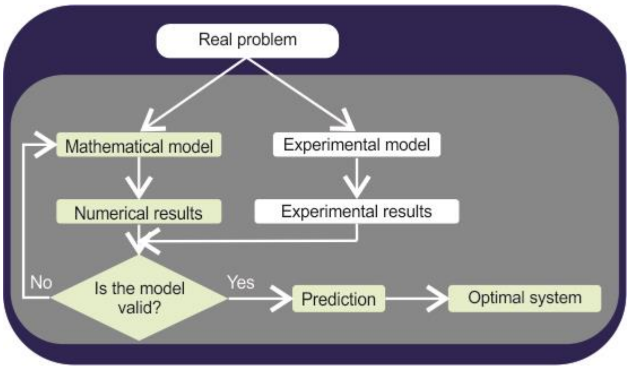


**Figure 1.** Scheme of the Systems Optimization Process

## 3. Application to the optimal design of sustainable infrastructures

This section is dedicated to illustrating how systems optimization can help in the design of sustainable infrastructures. For this, we show how three problems of great environmental interest can fit within this frame: the design of a river fishway for allowing fish to climb a dam, the design of a sustainable irrigation channel, and the management of an urban road network to minimize environmental pollution.

### *3.1. River fishways*

A fishway is a waterway to allow some fish species to pass by a dam or any other man-made obstruction in a river or stream (Prasetyorini et al, 2020). Usually, it consists of a sloping channel partitioned by weirs, baffles, or vanes with openings for fish to swim through (Katopodis, 1992). There are many different types, and they can be sophisticated (Fig. 2-left) or much simple (Fig. 2-right), but the objective is always the same: to get a velocity in the channel that enable fish to overcome the dam and continue upstream towards spawning. Therefore, it is necessary to simulate the water velocity in the channel before building the fishway. For a fixed geometry, this can be done using the well-known shallow water equations (Bermúdez, et al., 1991), which provide the height of the water and the velocity field in the channel over time. This model can be solved by finite difference, finite element or finite volume methods, and it can be validated by comparing the numerical results, for example, with those reported by Puertas et al. (2004) and Teijeiro-Rodríguez et al. (2006). For a fishway like the one studied in Teijeiro-Rodríguez et al. (2006), Fig. 3-left shows the velocity field in the central pool obtained by solving the shallow water equations with a mixed implicit finite element method. In this figure can be clearly observed the two large recirculation areas at both sides of the slot, which has been previously analyzed in the experimental work of Teijeiro-Rodríguez et al. (2006).

**Figure 2.** Example of two fishways: in the Broad River (USA), on the left -Credit: City of Columbia-, and in the Ouro River (Spain), on the right

Just as discussed in previous section, once this model has been validated, it can be used for predicting and designing an optimal fishway. The problem consists of determining the location of the baffles which provides an optimal velocity field: we seek to minimize the eddy flow (to prevent fish from becoming disoriented) and to obtain a velocity suitable for fish leaping and swimming capabilities in the region near the slots, and one as low as possible in the remaining of the channel (to facilitate the rest of the fish). From the previous model (shallow water equations), this optimal design problem can be formulated and solved in the framework of optimal control theory (see Alvarez-Vazquez et al., 2012 and therein references). For the standard vertical slot fishway we are working with, Fig.

3-right shows the optimal configuration of the baffles in the central pool and the corresponding velocity field. As can be clearly seen, this configuration is much better for fish than the initial one, shown in Fig. 3-left.

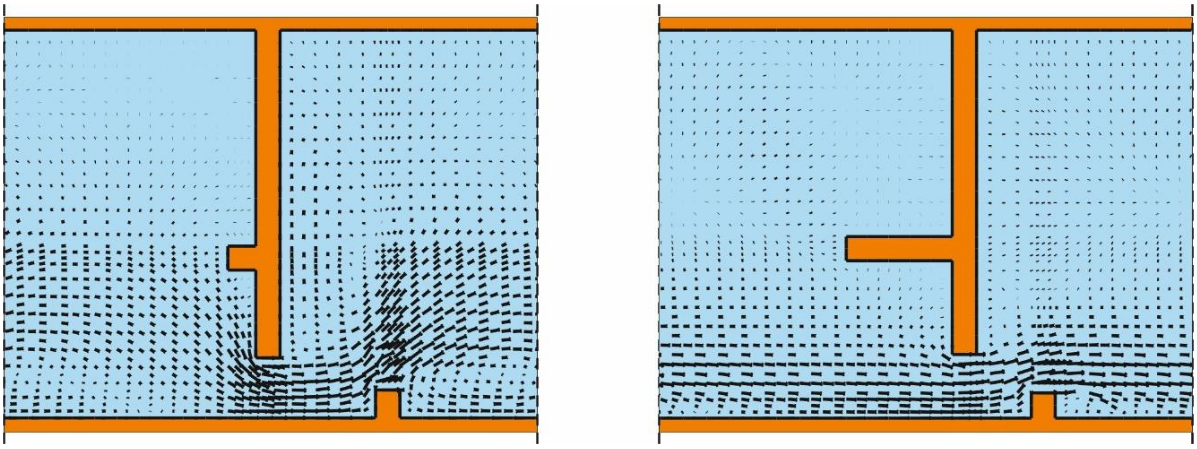

**Figure 3.** Velocity field in the central pool of a standard vertical slot fishway, corresponding to an initial configuration (left) and to the optimal design (right)

*3.2. Irrigation channels*

**Figure 4.** Joi-Pur irrigation channel in Bar-Sultan Pur village of Nangarhar Province (Afghanistan) -Credit: Rumi Consultancy/ World Bank-

Crops need water to grow and in arid regions, irrigation channels (Fig. 4) are the conveyance mechanism to provide water. The main problem that this infrastructure suffers is the sedimentation and erosion process, which is responsible for the instability in canals for surface drainage, for the loss of channels capacity and, in general, for the malfunction of irrigation systems. The issue is to determine the geometry of the channel to minimize the effects of this sedimentation/erosion process.

To simulate the sedimentation dynamic in a channel is necessary to couple the shallow water equations, which provide, just as we have said above, height of water and velocity field, with the equations for sediment transport, which provide concentration of suspended particles and height of sediment on the bottom. This system can be solved by different numerical techniques which have already been implemented in commercial software packages (see, for instance, MIKE21, 2001). From this numerical simulation tool, the optimal design of an irrigation channel can be formulated and solved in the framework of numerical optimization (see Alvarez-Vázquez et al., 2018b). A good channel design can help to reduce the undesired effects of sedimentation process. For example, Fig. 5 shows how the effect of erosion in a trapezoidal channel is much lower when the geometry is optimized.

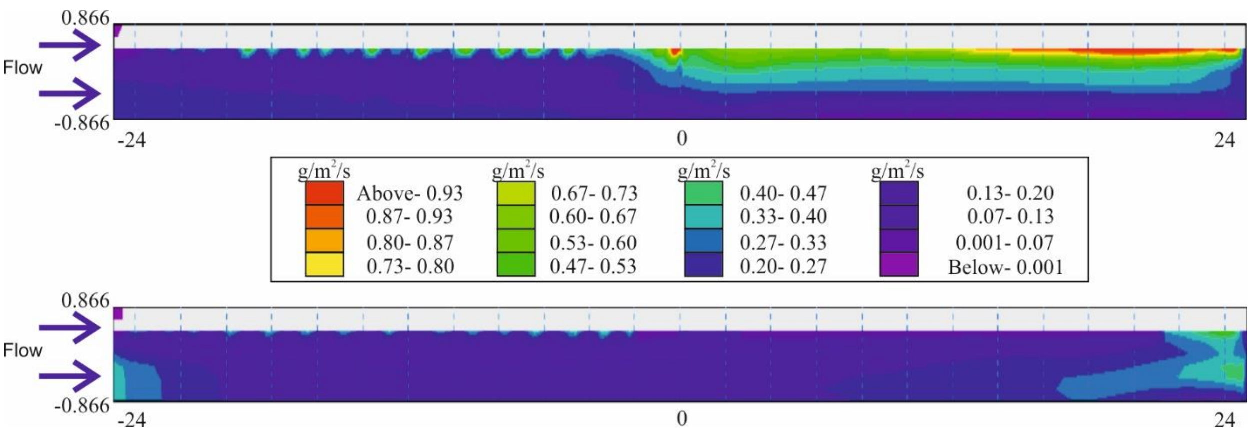


**Figure 5.** Erosion on the bottom of a trapezoidal channel, corresponding to a random initial configuration (up) and to the optimized geometry (down)

*3.3. Urban road networks*

The management of traffic flow on cities is a very important problem for municipal governments and almost all big cities suffer congested traffic problems. Linked to this overcrowded traffic situation is the problem of air pollution, which arrives with anticyclonic situations (with low winds and temperature inversion) and causes the very famous *pollution beret* (Fig. 6), one of the most environmental problems nowadays.

**Figure 6.** Pollution beret on Madrid (Spain) -Credit: lahoradigital.com-

Under this panorama, the problem is to simulate the air pollution due to traffic flow and design an urban road network which helps to minimize these undesired pollution episodes. Recently, Alvarez-Vázquez et al. (2017) proposed a mathematical model to simulate air pollution, by combining the 1D Lighthill-Whitham-Richards traffic model for road networks with a classical 2D advection-diffusion-reaction pollution model for the atmosphere. From this model, some problems have been studied to mitigate air pollution in the framework of systems optimization: the management of an urban road network from cooperative (Vázquez-Méndez et al., 2019) and hierarchical perspectives (García-Chan et al, 2022), the expansion of the network under an environmental viewpoint (Alvarez-Vázquez et al, 2018), and the design of an ecologically road corridor in which a new highway will be planned. This type of environmental problems is very complicated, and systems optimization is shown as a very useful tool for helping in the decision-making process. However, there are many aspects that can influence the simulation process and results obtained so far must be taken with great caution, and there is still a long way to go.

## 4. Conclusions

Systems optimization, widely used to minimize costs in industrial processes, can be also very useful in the design of sustainable infrastructures, where ecological and environmental objectives are considered. This complex methodology has been successfully applied to realistic situations, but to apply it in real life cases, close collaboration between mathematicians and environmental and civil engineers is vital. We hope that this can be carried out in the near future.

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