

An Advanced Boussinesq-Type Model for Wave Propagation in Coastal and Harbour Areas

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Abstract: In this paper, an advanced numerical model for the simulation of wave propagation and transformation in coastal areas and inside ports is presented. This model is a fully dispersive and highly nonlinear 2DH Boussinesq-type model, extended to cover real-life applications, capable of simulating the transformation of complex multi-directional irregular wave fields in coastal and harbour areas with varying bathymetries. This version of the model is coupled with wave overtopping empirical formulae as described in the EurOtop (2018) manual. The numerical model when compared with experimental data showed that a more than satisfactory agreement was achieved in most cases. A real case study, in the coastal zone of Rethymno in the Island of Crete, Greece, including the current port infrastructure, is also presented.

Keywords: Boussinesq-type model, coastal processes, irregular multi-directional waves, numerical simulation, wave overtopping.

1. Introduction

Simulation of nearshore wave propagation is of paramount importance in port and coastal engineering projects. Wind waves are one of the most important driving factors in the coastal environment. Understanding wave hydrodynamics and the processes induced by them is important for designing marine structures and for managing of the coastal zone. During the past few decades, numerous researchers have contributed to the development of numerical models capable of simulating complex physical phenomena such as wave shoaling, diffraction, refraction, and breaking.

Boussinesq-type wave models incorporate highly nonlinear wave characteristics partially disregarding the original Boussinesq assumptions and simulate fully dispersive properties simulating with high accuracy the wave climate in a coastal area and the wave tranquility inside a port. However, a serious restriction of the Boussinesq-type models is that they lose some of their efficiency when simulating wave propagation in deep waters with various researchers attempt to eliminate the water depth limitation.

In this paper the evolution of an advanced numerical model for the simulation of wave propagation and transformation in coastal areas around and inside ports and harbours is presented. The initial version of the model, developed by Chondros and Memos (2014), was capable of simulating non-breaking and breaking long and short crested waves in a variety of bottom profiles and structures, from very deep through to shallow waters. The applicability of that model was further extended by Metallinos et al. (2019), in order to tackle wave propagation over submerged porous breakwaters.

The main goal of this research is to present an extended version of the model (called BSQ hereinafter) covering real-life applications that include multi-directional irregular wave generation and estimation of the wave overtopping discharge at coastal structures. The latter is based on the empirical formulae as described in the EurOtop (2018). The model results are compared with the experimental measurements and showed that an overall satisfactory agreement was achieved. To this end, the study area of Rethymno in the Island of Crete, Greece is examined herein. Major coastal flood events have been encountered throughout the years, mainly due to combined effects of storm surge and wave overtopping, resulting in severe damages and posing a serious threat for coastal residents.

2. Model Description

BSQ model is a sophisticated numerical model which was initially developed by Chondros and Memos (2014) based on the formulation of Madsen and Schäffer (1998). This model was capable of simulating non-breaking and breaking long and short crested waves in a variety of bottom profiles and structures of steep slopes, extending the applicability from very deep to shallow waters and thus overcoming a shortcoming of most models of the same type. The applicability of that model was further extended by Metallinos et al. (2019), in order to tackle wave propagation over submerged porous breakwaters, as well as to estimate the velocity distribution inside the structure body, taking into account its porosity. The model's equations read in 2DH form:

$$\zeta_t + \nabla[(d + \varepsilon\zeta)\mathbf{U}] = 0 \quad (1)$$

$$\begin{aligned} & \mathbf{U}_t + \nabla\zeta + \frac{1}{2}\varepsilon(\mathbf{U} \cdot \nabla) \\ & + \mu^2(\Lambda_{20}^{III} + \varepsilon\Lambda_{21}^{III} + \varepsilon^2\Lambda_{22}^{III} + \varepsilon^3\Lambda_{23}^{III}) \\ & + \mu^4(\Lambda_{40}^{III} + \varepsilon\Lambda_{41}^{III}) + O(\mu^6, \varepsilon^2\mu^4) = 0 \end{aligned} \quad (2)$$

where ζ is the surface elevation, $\mathbf{U} \equiv (U, V)$ is the depth-averaged horizontal velocity vector, $\nabla \equiv (\partial/\partial x, \partial/\partial y)$ is the gradient operator, d is the water depth above the structure, ε is the nonlinearity parameter equal to H/d (where H is the local wave height) and μ is the frequency dispersion parameter equal to h/L (where L is the local wavelength). For the Λ^{III} terms the reader is referred to the original paper of Madsen and Schäffer (1998) and to that of Chondros and Memos (2014).

Wave overtopping is incorporated into the BSQ wave model through a computationally efficient approximation/methodology to calculate the mean wave overtopping discharge at the lee of a structure. The distinct steps of this approximation/methodology have as follows:

- The type of the structure (smooth sloping or vertical wall) is defined
- Geometric parameters of the structure (such as slope, crest freeboard) are specified
- Wave height values are extracted from the Boussinesq wave model at the toe of the structure
- Depending on the type of the structure the mean overtopping discharge is ultimately calculated

For a smooth sloping structure, the mean overtopping discharge is calculated through the following relationship (Eurotop, 2018):

$$\frac{q}{\sqrt{gH_{mo}^3}} = a \cdot \exp \left[- \left(b \frac{R_c}{H_{mo}} \right)^c \right] \quad (3)$$

where $a = 0.09 - 0.01(2 - \cot \alpha)^{2.1}$ for $\cot \alpha < 2$ and $a = 0.09$ for $\cot \alpha \geq 2$, $b = 1.5 + 0.42(2 - \cot \alpha)^{1.5}$, with a maximum of $b = 2.35$ and $b = 1.5$ for $\cot \alpha \geq 2$ and $c = 1.3$

For a vertical structure the mean overtopping discharge is calculated through the following relationship (Eurotop, 2018):

$$\frac{q}{\sqrt{gH_{mo}^3}} = 0.047 \cdot \exp \left[- \left(2.35 \frac{R_c}{H_{mo} \gamma_f \gamma_\beta} \right)^{1.30} \right] \quad (4)$$

The empirical formulations presented above can provide a reliable and efficient estimation of mean overtopping discharge at the lee of every structure configuration especially when combined to an accurate Boussinesq wave model treating the propagation of random multi-directional waves in the nearshore.

Irregular waves are generated in the model specifying either a Jonswap, Pierson-Moskowitz or TMA spectrum function, and dividing it into discrete wave components utilizing the single-summation method of Yu et al. (1991). In this method each unique frequency component can only propagate in one direction, while several wave components are able to propagate in the same direction.

3. Verification with Experiments

Numerical results of both models, comprising wave energy spectra, wave heights and wave overtopping discharges are compared against experimental data. The scenarios used for the comparison include the experimental layouts of Metallinos et al. (2019), Vincent and Briggs (1989) and Williams et al. (2019).

3.1. Experiment of Metallinos et al. (2019)

In this section, results of BSQ model compared to the measurements of Metallinos et al. (2019) are shown. The latter carried out 2DH laboratory experiments in a wave basin at the Hydraulic Engineering Laboratory, University of Patra. Part of these tests included measurements of free surface elevation over a permeable SB with steep slopes, under regular and irregular (Jonswap) wave attack, including breaking or non-breaking events. A permeable submerged breakwater (SB) with porosity $\varphi=0.50$ was constructed and placed on a mild sloping bottom at 1:15. The height of the structure was 0.2 m at the middle of the crest, while the crest width was 0.5 m. The SB was made of natural stones with $d_{50}=0.05$ m sloping 1:2 at both sides and downstream was placed sand in order to study the evolution of bed morphology. The water depth at the SB axis was 0.25 m, leaving a 0.05 m freeboard below S.W.L. The validation shown here, consists of an irregular unidirectional wave with $H_s = 0.080$ m and $T_p=2.00$ s. The comparison between the measured and the computed wave energy spectra at two stations are given below.

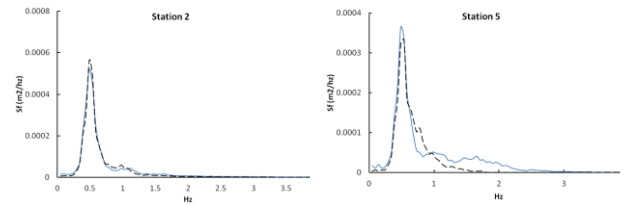


Figure 1 Comparison of the BSQ model spatial evolution of wave spectrum (dot line) against experimental data of Metallinos et al. (2019) (blue line).

From the above comparisons of the model's results with the measurements as well as the results of other cases not shown here, it can be deduced that the model was able to describe accurately the wave field over and around a porous SB with steep slopes capturing the nonlinear phenomena at the free surface, under random wave attack.

3.2. Experiment of Vincent and Briggs (1989)

Vincent and Briggs (1989) performed experiments on random waves with frequency-direction spreading which pass over a submerged elliptic shoal by using a directional spectral TMA spectral wave generator in a basin of the Coastal and Hydraulics Laboratory of the U.S. Army Engineer Research and Development Center. Here, two irregular wave scenarios, i.e. a) Case U3: Unidirectional wave and b) Case N3: Multi-directional wave with spreading $\sigma_m = 10^\circ$, both of them with $H_s = 0.0254$ m and $T_p = 1.3$ s are presented: For the numerical simulations of

this experiment, the bathymetry with a submerged elliptic shoal is reproduced. The computational grid was $\Delta x = \Delta y = 0.05$ m and the timesteps $\Delta t = 0.01$ s. The model run for 30 s and the last 5 periods are used to extract the numerical results of wave heights.

Below the comparison of the BSQ model against the experimental data are shown at S4 transect. An overall satisfactory agreement of the model results with the experimental measurements is a gain noted.

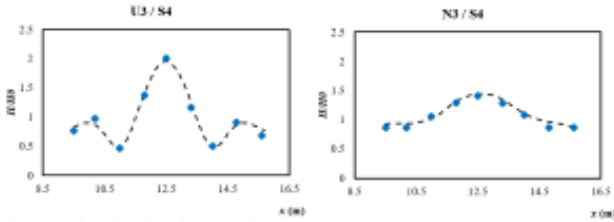


Figure 2 Comparison of related waves heights of the BSQ model (dot line) against experimental data of Vincent and Briggs (1989) (blue circles).

3.3. Experiment of Williams et al. (2019)

The capability of the Boussinesq wave model to calculate wave overtopping discharges induced by a random wave field was validated by reproducing the experiments of Williams et al. (2019). The latter conducted physical model tests of various incident wave conditions in a flume 15 m long, 0.23 m wide with an operating depth up to 0.22 m with two structure configurations; a smoothly sloping one and a vertical wall. Four wave conditions for each structure configurations were tested, i.e. wave scenarios TS01-SS to TS04-SS for the smoothly sloping structure and wave scenarios TS01-VW to TS04-VW for the vertical wall respectively.

The overtopping discharge was calculated for each test, with the results plotted using the dimensionless parameters Q^* , where $Q^* = \frac{q}{\sqrt{gH_{mo}^3}}$. More information about the layout

of the experiments and wave conditions tested can be found in Williams et al. (2019). In Table 1 the comparison of the model results with the experiments are given.

Table 1. Comparison of measured dimensionless overtopping discharge (Q^*) against experimental data of Williams et al. (2019)

Wave cases	Q^* (measured)	Q^* (calculated)	Q^* (Diff %)
TS01-SS	0.00865	0.00689	20%
TS02-SS	0.00536	0.00384	28%
TS03-SS	0.00233	0.00139	40%
TS04-SS	0.00054	0.00024	56%
TS01-VW	0.00457	0.00339	26%
TS02-VW	0.00427	0.00281	34%
TS03-VW	0.00219	0.00149	32%
TS04-VW	0.00082	0.00059	28%

From the above, an overall satisfactory agreement of the model results with the experimental measurements is noted showing that this simple technique by combining the wave results of the Boussinesq-type model with the empirical

formulations of EurOtop, 2018, are able to predict the overtopping discharges at coastal structures becoming thus a reliable tool for the coastal engineering community.

4. Application Study

Implementation of the BSQ model is carried out in the coastal zone of Rethymno in the Island of Crete, Greece, including the port of Rethymno. The bathymetric grid was constructed, based on executed surveys, with spatial step of 2.5 m as shown in Figure 3. In the same figure transects that are used to calculate the wave overtopping discharges can also be found.

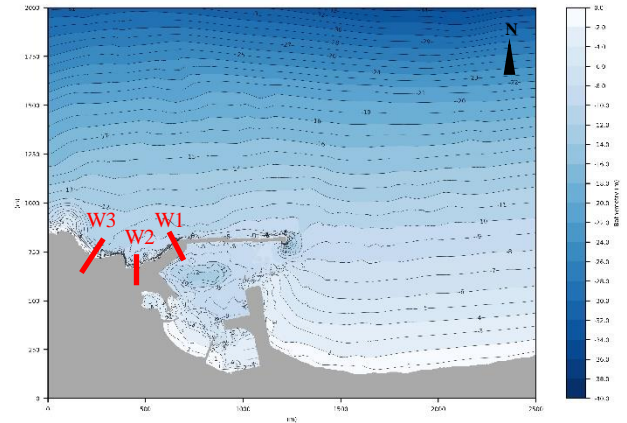


Figure 3 Bathymetry chart of the coastal zone of Rethymno, Crete, Greece. Port of Rethymno is included in the study area

Numerical simulations of wave transformation from offshore to nearshore were executed by implementing the BSQ model. Incident multi-directional irregular waves with spreading $\sigma_m = 10^\circ$ were considered with $H_s = 4.80$ m, $T_p = 10.15$ s and MWD 0.82° (from N). This scenario is referred to a past extreme event occurred in 10/12/2010. More information about the wave offshore data can be found in the companion paper of Chondros et al. (2021).

The 2DH numerical domain in the model was discretized using a grid size of $\Delta x = \Delta y = 2.5$ m and a time step of $\Delta t = 0.01$ s while the total simulation time was 600 s. The results are illustrated in maps below (Figures 2 and 3) depicting steady state conditions of wave heights and snapshot of surface elevations.

It can be observed that the model can adequately simulate the wave processes of wave breaking, shoaling/refraction, diffraction and reflection. Regarding reflected wave trains, an increase in wave height is observed windward the breakwater. Note that the wave-wave interactions were taken into account due to the improved dispersion characteristics and high non-linearity embedded in the model. The proper predicted reflected height is of paramount importance in coastal engineering studies with the presence of coastal structures and in wave agitation studies due to the reflected nature of a port basin.

The wave height results, after achieving steady-state conditions, are shown in Figure 4 while a snapshot of surface elevation results are shown in Figure 5. Having determined the nearshore wave characteristics at the toe of the structures the wave overtopping discharges are

calculated by the couple model in three indicative transects (W1-W3) as shown in Table 2.

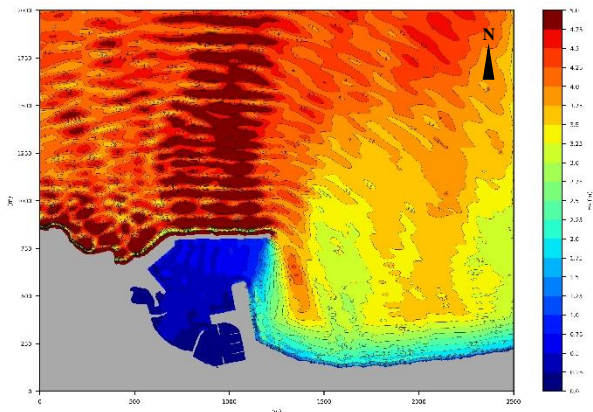


Figure 4 Wave height results of BSQ model in the coastal zone of Rethymno

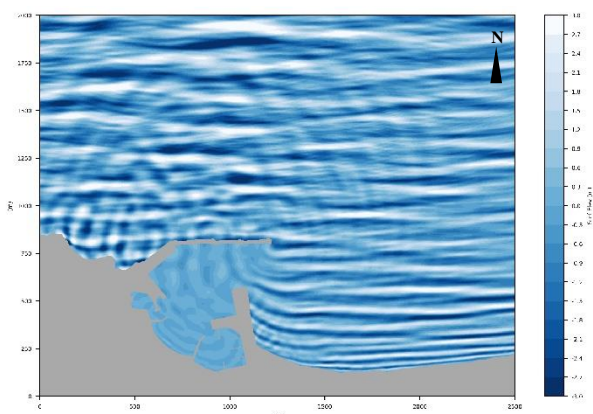


Figure 5 Surface elevation results of BSQ model in the coastal zone of Rethymno

Table 2. Nearshore wave height at the toe of the structures and mean wave overtopping discharges

	Transect	Event
Nearshore Wave Height H_s (m)	W1	4.75
	W2	4.90
	W3	4.53
Overtopping discharge q (l/s/m)	W1	37.54
	W2	230.92
	W3	6.52

5. Conclusions

In this paper, an advanced fully dispersive and highly nonlinear 2DH Boussinesq-type model for the simulation of wave propagation and transformation in coastal areas and inside ports is presented. This model is extended to simulate the transformation of complex multi-directional irregular wave fields in coastal and harbour areas. Also, it is coupled with wave overtopping empirical formulae as described in the EurOtop (2018) manual in order to predict the overtopping discharge at port and coastal structures.

The numerical model when compared with test data showed that satisfactory agreement was achieved. Given the satisfactory agreement between models' results and laboratory measurements, the model was successfully applied to an application study in the coastal zone of

Rethymno, Crete, Greece and it was able to reproduce adequately phenomena related to the physical processes in a coastal and harbour area. Mean wave overtopping discharges were also calculated in selected transects.

Based on the comparison results with the laboratory measurements and the results of the application study, the proposed model is deemed to constitute a suitable model for the design and evaluation of the wave propagation in a coastal area, wave disturbance into a port basin and wave overtopping in front of coastal structures addressing thus significant needs of the engineering community and becoming a useful tool for a coastal engineer.

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