

# Effect of the dependence structure and time irreversibility of streamflow in flood inundation mapping with focus on the long-term persistence behaviour

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**Abstract.** The marginal structure of streamflow, with focus on the right tail behaviour, is considered as the main factor in flood risk assessment, while little is known on the effect of the temporal dependence structure and irreversibility of streamflow. Interestingly, the second-order dependence behaviour of streamflow is shown to highly deviate from a white noise behaviour (i.e., temporal independence), and to rather exhibit a Hurst-Kolmogorov (HK) behaviour, with strong autocorrelation and irreversibility at small scales and long-term persistence at large scales. The HK dynamics is known to be characterized by large uncertainty and variability, and therefore, it is expected to have a non-negligible impact on flood inundation mapping, especially in cases of successive storm events. Through benchmark experiments and real case scenarios, we investigate the influence of these effects in several output features of flood risk modelling such as flood depth, velocity, and duration, and we discuss possible consequences for insurance policies.

**Keywords:** stochastics, flood, streamflow, dependence, irreversibility

## 1. Introduction

The time-irreversibility is a model attribute that can efficiently simulate time causal relations, and it has been shown that in atmospheric processes irreversibility mostly exists at the finest scales, while may be negligible at hydrologically relevant time scales (Koutsoyiannis, 2019), with the exception of the irreversibility of streamflow that can be marked even for several days (Vavoulogiannis et al., 2021).

The long-range dependence (LRD; also known as long-term persistence or the Hurst phenomenon; Hurst, 1951; earlier and independently analyzed by Kolmogorov, 1940) is a model attribute characterized by the power-law drop of a process's second-order dependence structure at large scales. It is known that the LRD model cannot be applied at the finest scales of a process (Koutsoyiannis et al., 2018), where a fractal behaviour is often apparent (Gneiting and Schlather, 2004), and with a transient behaviour identified at the intermediate scales of various

hydrometeorological processes (Dimitriadis and Koutsoyiannis, 2018). The combination of the aforementioned mixed fractal-transient-LRD behaviours at the scale domain, is known as the Hurst-Kolmogorov dynamics (Koutsoyiannis, 2010), and has been shown to efficiently simulate the dependence structure from the finest to the largest available scales of a large variety of natural processes, including streamflow (Dimitriadis et al., 2016; 2021).

Since, both the above behaviours (i.e., time-irreversibility and HK) are apparent in the streamflow process, it is argued that will also have a non-negligible impact on stochastic processes involved in flood inundation mapping, such as flood depth and water velocity. While the effect of time-irreversibility can be quantified by the skewness of the differenced (depth and velocity) processes (Koutsoyiannis, 2020), the reflection of the LRD (and in general, the HK) behaviour at the flood inundation mapping cannot be easily identified by the common block maxima method for extremes (Iliopoulou and Koutsoyiannis, 2018), but rather by indexes indicative of the flood event temporal duration and the cluster size of depth and velocity caused by successive storm events (see preliminary application in Dimitriadis and Koutsoyiannis, 2019).

In this work, we investigate the influence of the above characteristics on the flood inundation mapping by quantifying the mean temporal cluster size of cell values (depth and velocity) over threshold as well as the skewness of the differenced flood depth and water velocity, from a benchmark application at the Peneios river in Greece (Fig. 1). It is stressed that both the LRD and time-irreversibility behaviours are seldom taken into account in stochastic simulations (Koutsoyiannis and Dimitriadis, 2021) and insurance policies (Papoulakos et al., 2020); however, as we show below, they may be proven crucial for the flood risk assessment.

## 2. Methods and Results

The LRD behaviour can be robustly visualized and quantified through the estimated variance in the scale

domain. It is noted that this stochastic metric (i.e., variance of the averaged process vs. scale) was loosely applied in the 90s, for later to be disproved as a weak estimator of the LRD (e.g., Beran, 1988; see discussion in Dimitriadis et al., 2021). However, after linking it to its theoretical value (e.g., Papoulis, 1991) and by properly adjusting for statistical bias (Koutsoyiannis, 2011), it was shown to be an even more advanced estimator for LRD than the common metrics of autocovariance in the lag domain and the power-spectrum in the frequency domain (Dimitriadis and Koutsoyiannis, 2015). Since a single name for this method did not exist (as, for example, for the periodogram or the correlogram methods), Koutsoyiannis (2010) coined the term ‘climacogram’ to emphasize the graphical representation and the link of the concept to scale (i.e., climax in Greek). The climacogram estimator can be expressed as (Koutsoyiannis, 2021):

$$\hat{\gamma}(\kappa\Delta) = \frac{\kappa}{n} \sum_{i=1}^{n/\kappa} \left( \underline{x}_i^{(\kappa)} - \hat{\mu} \right)^2 + \gamma(n\Delta) \quad (1)$$

where  $\kappa = k/\Delta$  is the dimensionless scale and  $k$  the temporal scale,  $\Delta$  is the time resolution of the process (e.g., 1 day),  $n$  is the length of the timeseries, and  $\underline{x}_i^{(\kappa)}$  is the  $i$ -th element of the averaged sample of the process of interest  $\underline{x}$  at scale  $\kappa$  (i.e.,  $\underline{x}_i^{(\kappa)} = \frac{1}{\kappa} \sum_{j=(i-1)\kappa+1}^{i\kappa} \underline{x}_j$ ), and  $\gamma(\kappa\Delta)$  is the climacogram model adopted here for the streamflow process at Peneios river (Fig. 2), i.e.,

$$\gamma(k) = \frac{q}{(1+k/\Delta)^{2-2H}} \quad (2)$$

where  $q$  is the variance at scale zero, and  $H$  is the Hurst parameter (i.e., for  $0.5 < H < 1$  the process exhibits LRD behaviour, while for  $0 < H < 0.5$  an anti-persistent behaviour, and for  $H = 0.5$  a white-noise behaviour).

Also, for the marginal structure of the streamflow, the Pareto-Burr-Feller distribution (also known as Pareto IV or Burr XII, without however giving recognition to all contributors; see discussion in Koutsoyiannis et al., 2018) is adopted for the application (Fig. 2):

$$F(x) = 1 - \left( 1 + \xi\zeta \left( \frac{x}{\lambda} \right)^\xi \right)^{-\frac{1}{\xi\zeta}} \quad (2)$$

where  $\lambda$  is a scale parameter, and  $\xi, \zeta$  are shape parameters.

For the visualization of the impact of the time-irreversibility and mean cluster size: (a) a white-noise (WN) process (i.e.,  $H = 0.5$ ) is simulated with the same marginal structure of the observed timeseries but with time-reversibility; (b) a process without time-irreversibility (TR) is simulated with the same climacogram model and PBF marginal distribution applied for the observed timeseries; and (c) a time-irreversible (TI) process is simulated with the same climacogram and PBF models. It is noted that the seasonal periodicity apparent in the observed timeseries is only illustrated in Figure 2, and not used for the comparison between the time-symmetrical and white-noise timeseries, since we are not interested here in the seasonality impact on the LRD and time-irreversibility (such comparisons will be performed in future works).

For the above simulations, the Axisymmetric Moving Average (ASMA) scheme (Koutsoyiannis, 2020) is followed, where the observed skewness coefficient of the timeseries and the differenced timeseries is estimated as 3.9 and 1.3, respectively. Therefore, the skewness ratio, defined as the skewness of the differenced process standardized by the skewness of the regular process, is estimated as  $1.3/3.9 = 0.3$ , while for the selected simulations is estimated as  $\sim 0.0$  for both the simulation of white-noise and the one with no time-irreversibility, and 0.4 for the simulation with time-irreversibility.

For the flood inundation, the HEC-RAS 2D ([www.hec.usace.army.mil/software/hec-ras/](http://www.hec.usace.army.mil/software/hec-ras/)) model is used with the dynamic-wave scheme, and with a rectangular grid of  $50 \times 50$  m<sup>2</sup> pixel size.

In Figure 1, we illustrate a flood inundation map at a steady 800 m<sup>3</sup>/s streamflow. In Figure 2, we compare the observed (OB) timeseries with the simulated timeseries adjusted for time-irreversibility (TI), while also showing the simulation without time-irreversibility along with the white-noise (WN) simulation.

In Figure 3, we depict the mean cluster size (averaged over all the inundated cells) of the flood depth and water velocity over various thresholds equal to percentiles ranging from 0.1 to 0.9. It is noted that for the identification of the impact of the LRD behaviour the mean cluster sizes of the observed and simulated timeseries are standardized with the simulated white-noise timeseries.

Finally, for the identification of the impact of the time-irreversibility the skewness coefficient of the differenced timeseries is standardized with the regular skewness coefficient. The average (over all inundated cells) of this metric is estimated for the flood depth of the WN, TR, and TI timeseries, as -1.3, 2.2 and 7.1, respectively; and for the water velocity of the same timeseries as 0.2, 0.2, and 1.3, respectively.

### 3. Discussion and Conclusions

In this work, we show how the long-range dependence and time-irreversibility can highly affect the flood risk in terms of the flood depth and water velocity persistence, by performing benchmark simulations of over 10-years length of streamflow at the Peneios river in Greece.

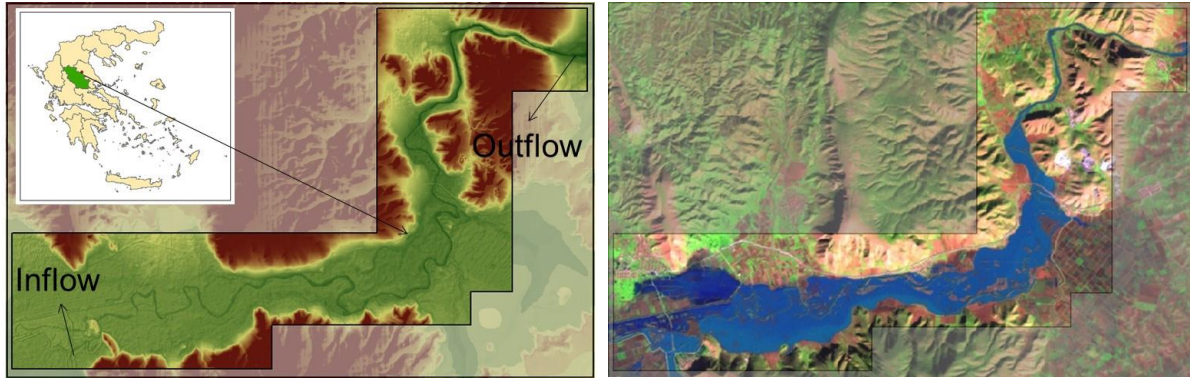
Specifically, it is shown that the cluster size is almost twice as high for the depth and velocity, when simulating the long-term persistence with a Hurst parameter of  $H = 0.8$ , as compared to the white noise behaviour. Also, it is found that there seems to be no effect of the time-irreversibility on the cluster size of the above two flood inundation processes (i.e., depth and velocity).

Although time-irreversibility is not reflected on the cluster size, it is shown to have a great impact on the skewness ratio, defined as the skewness of the differenced processes standardized by the skewness of depth and velocity. Particularly, the skewness ratio of velocity seems to be lightly affected by the flood inundation model in case of the white-noise and time-symmetrical timeseries (i.e., from 0.0 is increased to 0.2), while is particularly increasing at

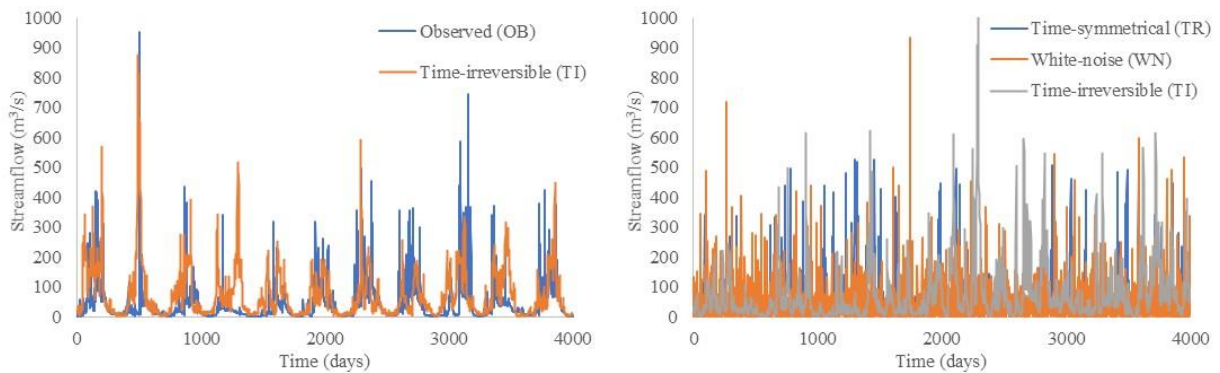
the time-irreversible timeseries (i.e., from 0.4 to 1.3). This increase can be explained by the diffusion generated by the flood model. In case of the flood depth, the skewness ratio is always heavily increasing for the LRD simulation regardless of time irreversibility (i.e., from 0.0 to 2.2, and from 0.4 to 7.1, respectively). The skewness ratio of the white-noise timeseries is heavily decreasing probably due to the fact that the depth (in contrast to the velocity) may

exhibit negative skewness coefficient, depending on the topography of the area and the flood inundation.

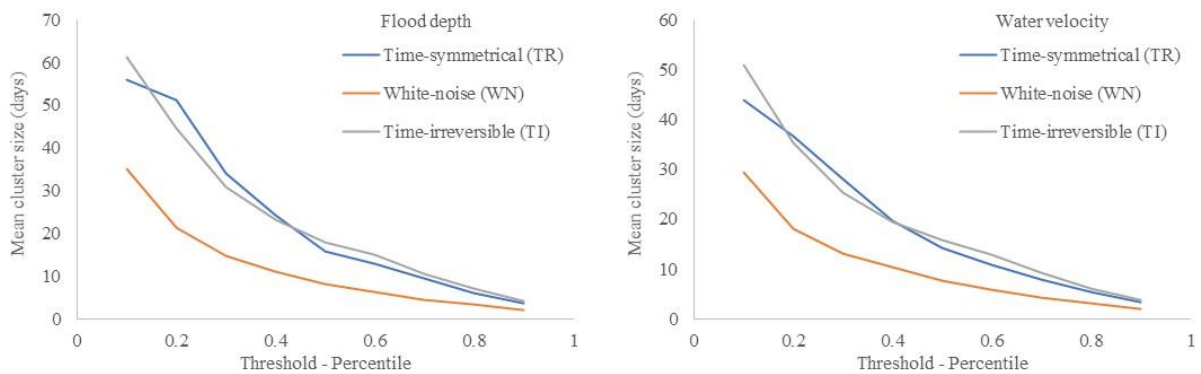
The above observations highlight the need for further investigation of the impact of the LRD and time-irreversibility behaviours in flood risk assessment.



**Figure 1.** The inflow-outflow positions and the flood simulation area [left], and the flood inundation map for a steady streamflow of  $800 \text{ m}^3/\text{s}$  [right], at the Peneios river in Thessaly plain in Greece [Source: Dimitriadis et al., 2018].



**Figure 2.** The observed (OB) and simulated with time-irreversibility (TI) timeseries with seasonality [left], and the simulated with (TI) and without (TR) time-irreversibility, and white-noise (WN) timeseries without seasonality [right].



**Figure 3.** The mean cluster size (in days) of the flood depth [left] and water velocity [right], for the simulated timeseries.

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